International Journal of Heat and Fluid Flow 30 (2009) 331–338

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

Turbulence modulation in dilute particle-laden flow

M. Mandø [a,*], M.F. Lightstone [b], L. Rosendahl [a], C. Yin [a], H. Sørensen [a]

[a] *Institute of Energy Technology, Aalborg University, Pontoppidanstræde 101, 9220 Aalborg, Denmark*
[b] *Department of Mechanical Engineering, McMaster University, 1280 Main Street West, Hamilton, Ontario, Canada L8S 4L7*

ARTICLE INFO

Article history:
Received 19 June 2008
Received in revised form 8 December 2008
Accepted 9 December 2008
Available online 19 January 2009

Keywords:
Turbulence modulation
Gas-particle flow
Eulerian–Lagrangian
Particle-laden pipe flow
K–ε model
Dilute flow

ABSTRACT

A new particle source term to account for the effect of particles on the turbulence equations based on the Euler/Lagrange approach is introduced and compared with existing models and experimental data. Three different sizes of particles are considered to cover the range of large particles, where augmentation of the carrier phase turbulence is expected, and small particles, for which attenuation is expected. The new model is derived directly from the balance equations of fluid flow and represents a combination of the so-called standard and consistent approaches. The performance of the new model surpasses that of the standard and consistent models and it is able to predict both the suppression and enhancement of fluid turbulence for small and large particles.

1. Introduction

Particle-laden flows have many important engineering applications such as pneumatic transport, combustion of pulverized fuels, dispersion of pollutants and spray drying. In each of these cases a fundamental understanding of the underlying phenomena which are responsible for the complex interaction between the particulate phase and the turbulent carrier flow is required to improve the design of engineering devices in which these flows occur. In the context of Computational Fluid Dynamics, Crowe et al. (1977) provided the PSI-CELL procedure for the momentum coupling between particles and the carrier phase. However, there is no consensus towards the influence of particles on the turbulence equations and no model has so far been able to reproduce the entire spectrum of experimental measurements. Experimental observation suggests that small particles tend to attenuate the carrier phase turbulence while large particles tend to augment the turbulence (Gore and Crowe, 1989). Furthermore, the magnitude of the change has been shown to scale with the particle concentration[1] (Kenning, 1996). The fundamental mechanisms which are most often associated with turbulence modulation are that the wake of particles is responsible for the additional production of turbulence while the particle-eddy interaction is responsible for the additional dissipation of turbulence (Yuan and Michaelides, 1992). For dense flows the effect of particle-particle collisions introduces another route for which turbulence modulation can proceed. Several models have been proposed to account for the influence of particles on the carrier phase turbulence. These can be divided into three distinct categories. The largest group of models derived the source term due to particles using the standard approach of Reynolds averaging used to derive the turbulence equations.

This results in a source term which always acts as sink for the turbulent kinetic energy and thus is only able to predict attenuation. This method has been labeled the "standard" approach (Lain and Sommerfeld, 2003). The turbulence equations for particle-laden flow can also be derived by considering that the instantaneous carrier phase velocity at the surface the particle must be equal to the particle velocity. This results in a term which for dilute flows is always positive and thus only acts to enhance the turbulent kinetic energy. This method is commonly referred to as the "consistent" approach (Lain and Sommerfeld, 2003). The last type of models can be referred to as semi-empirical or semi-heuristic. These are based on a mechanistic approach where additional source terms are defined as functional relationships of the wake size or other particle related parameters. In contrast to the standard and the consistent approach, models based on this approach are capable of predicting both attenuation and augmentation of turbulence. However, such an approach has been criticized for lacking rigor since the models are not derived from the balance equations of mass, momentum and energy, and thus cannot be introduced into conventional closure models without violating fundamental physical principles. The present work introduces a

* Corresponding author. Tel.: +45 9940 9248; fax: +45 9815 1411.
E-mail address: mma@iet.aau.dk (M. Mandø).

[1] In addition to the particle concentration, parameteres such as mass loading, volume loading, number concentration, mass fraction, volume fraction and interparticle spacing have been used specify the quantity of particles.

0142-727X/$ - see front matter © 2008 Elsevier Inc. All rights reserved.
doi:10.1016/j.ijheatfluidflow.2008.12.005

Nomenclature

C	constant [–]
l_e	integral length scale [m]
d_p	particle diameter [m]
g	gravity [m/s^2]
P	pressure [pa]
S	source [–]
u	velocity [m/s]
x	spatial coordinate [m]
t	time [s]
F	force [N]
V	volume [m^3]
m	mass [kg]
n	number of particles [–]
L_l	Lagrangian length scale [m]
Re	Reynolds number [–]
f	derivation from Stokes drag [–]
U	mean velocity [m/s]
k	turbulent kinetic energy [m^2/s^2]

Greek symbols

ε	dissipation rate [m^2/s^3]
ρ	density [kg/m^3]
φ	transport parameter [–]
Γ	diffusion coefficient [–]
σ	intensity, constant in k–ε model [–]
μ	viscosity [kg/m·s]
α	volume fraction [–]
τ	time scale/constant [s]

Superscripts

–	time average
′	fluctuating quantity

Subscripts

i,j	index
p	particle
u	momentum
k	turbulent kinetic energy
ε	dissipation rate
t	turbulent
0	clear flow
V	Stokes flow
c	clear flow

new derivation of the source term for particle-turbulence interaction consistent with the governing equations of fluid flow. The resulting source term represents a "hybrid" between the standard and consistent approaches and is capable of predicting both attenuation and augmentation. In this paper the new model has been compared to the experimental studies by Tsuji et al. (1984) and Kulick et al. (1994) as well as to a representative model for each of the aforementioned approaches.

2. Numerical approach

An Eulerian/Lagrangian approach is used to calculate the particle-laden gas flow. The continuous phase flow field is obtained from solution of the Reynolds averaged equations for fluid flow along with the k–ε turbulence model to achieve closure. The conservation equations for steady fluid flow, extended to take into account the presence of particles, is given as:

$$\frac{\partial \rho \bar{u}_j \phi}{\partial x_j} = \frac{\partial}{\partial x_j} \Gamma_\phi \frac{\partial \phi}{\partial x_j} + S_\phi + S_{\phi p} \tag{1}$$

Here ρ is the continuous phase density, $\bar{u}_j$ is the mean velocity components, φ represent the transported parameter, Γ_φ is a diffusion coefficient, S_φ is the usual fluid phase source term and $S_{\varphi p}$ is the source term due to the particles. These quantities are presented in Table 1. Here, μ is the gas phase viscosity, P is the mean pressure, g is the gravitational acceleration, k is the turbulent kinetic energy and ε is the dissipation rate.

The complete set of equations for the continuous phase is discretised using the upwind scheme and solved iteratively using the SIMPLE algorithm. In the present work the commercial solver Fluent has been used to perform the calculations. The particle trajectories are calculated using the following set of ordinary differential equations:

$$\rho_p V_p \frac{du_{pi}}{dt} = \frac{\rho_p V_p}{\tau_p}(u_i - u_{pi}) + V_p(\rho_p - \rho)g_i + F_i \frac{dx_{pi}}{dt} = u_{pi} \tag{2}$$

Here, u_{pi} are the instantaneous particle velocity components, x_{pi} are the coordinates of the particle position, ρ_p is the particle density, V_p is the particle volume, F_i represent forces other than drag and gravity and τ_p is the particle response time calculated using the following set of supporting equations:

$$\begin{aligned} &\tau_p = \tfrac{\tau_V}{f}, \qquad f = 1 + 0.15\mathrm{Re}^{0.687}, \\ &\tau_V = \tfrac{\rho_p d_p^2}{18\mu}, \quad \mathrm{Re} = \tfrac{\rho |u_{pi} - u_i| d_p}{\mu} \end{aligned} \tag{3}$$

where d_p is the particle diameter. Only the Saffman lift force is considered in addition to the drag and the gravity force and all other forces such as pressure gradient, virtual mass and Basset history force are considered negligible. The instantaneous velocity is obtained by adding the mean velocity to a fluctuating velocity component which is sampled from a Gaussian probability distribution function. The interaction time for which the sampled fluctuating velocity persists, is determined from the minimum of the eddy life time and the eddy crossing time which are calculated using appropriate time and length scales associated with the k–ε model. For more on the numerical approach, the specific schemes and models the reader is referred to the Fluent documentation (Fluent, 2006).

Table 1
Summary of terms and constants in the general equation.

φ	S_φ	Γ_φ
1	0	0
$\bar{u}_i$	$\frac{\partial}{\partial x_j}\left(\Gamma_{\bar{u}_i}\frac{\partial \bar{u}_j}{\partial x_i}\right) - \frac{\partial P}{\partial x_i} + \rho g_i$	$\mu + \mu_t$
k	$G_k - \rho\varepsilon$	$\mu + \mu_t/\sigma_k$
ε	$\frac{k}{\varepsilon}(C_{\varepsilon 1} G_k - C_{\varepsilon 2}\rho\varepsilon)$	$\mu + \mu_t/\sigma_\varepsilon$
$G_k = \mu_t\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_j}, \quad \mu_t = C_\mu \rho \frac{k^2}{\varepsilon}, \quad \sigma_k = 1.0$		
$C_\mu = 0.09, \quad C_{\varepsilon 1} = 1.44, \quad C_{\varepsilon 2} = 1.92, \quad \sigma_\varepsilon = 1.3$		

3. Effect of particles on continuous phase

The momentum source due to the presence of particles is found by examining the change in momentum of a particle as it passes through each control volume. By time and ensemble averaging for each control volume it can be expressed in the following form (Gouesbet and Berlemont, 1999):

$$\overline{S_{upi}} = n\left\langle m_p \left(\frac{du_{pi}}{dt} - g_i\right)\right\rangle \tag{4}$$

where n is the mean number of particle in the per unit volume, m_p is the particle mass and $\langle \ldots \rangle$ indicate mean values over all particle trajectory realizations. This expression has been implemented into Fluent whereas the influence on the turbulence equations need to implemented using custom source terms. In the literature three different approaches exist to derive appropriate source terms for the turbulence equations.

3.1. Standard approach

The source term for the standard approach is derived by multiplying the momentum equation by u_i and applying a Reynolds averaging procedure (Chen and Wood, 1985; Gouesbet and Berlemont, 1999; Lightstone and Hodgson, 2004). After subtracting the mean kinetic energy, an expression for the turbulent kinetic energy due to the presence of particles results as:

$$S_{kp} = \overline{u'_i S'_{upi}} \tag{5}$$

If only the drag force is considered this term can be expressed as:

$$S_{kp} = \frac{\alpha \rho_p}{\tau_p}\left(\overline{u'_i u'_{pi}} - \overline{u'_i u'_i}\right), \quad \overline{u'_i u'_i} = 2k \tag{6}$$

Eq. (6) is often referred to as being dissipative considering that the particles are accelerated by the motion of the fluid and thus the particle velocity u'_{pi} is smaller than the fluid velocity u'_i (Elghobashi, 1994). Usually, models based on this approach are only capable of predicting attenuation. Several authors have presented models for the unknown first term. Here we only present the most recent development of the standard approach. Thus by considering the crossing trajectory effect, the unknown correlation can be derived analytically as (Lightstone and Hodgson, 2004):

$$\begin{aligned} &\overline{u'_i u'_{pi}} = 2k\tfrac{\tau^*}{\tau^*+\tau_p}, \quad \tfrac{1}{\tau^*} = \tfrac{|u_i - u_{pi}|}{L_l} + \tfrac{1}{\tau_{L_l}} \\ &L_l = 2\tau_{L_l}\sqrt{2k/3}, \quad \tau_{L_l} = 0.135\tfrac{k}{\varepsilon} \end{aligned} \tag{7}$$

where τ_{Ll} and L_l is the Lagrangian time and length scale. The additional dissipation due to the particles, $S_{\varepsilon p}$, is assumed to be proportional to the similar terms in the k-equation. To get the right units each term is multiplied by ε/k:

$$S_{\varepsilon p} = C_{\varepsilon 3}\frac{\varepsilon}{k}S_{kp} \tag{8}$$

where the value of the constant $C_{\varepsilon 3}$ is suggested to be 1.1.

3.2. Consistent approach

Another approach, which provides what is commonly known as the consistent terms, starts with the mechanical energy equation for the fluid phase and subtracts the product of the mean velocity and the momentum equation to obtain an expression for the turbulent kinetic energy (Crowe, 2000). The source term due to the presence of the particles is then given as:

$$S_{kp} = \overline{u_{pi} S_{upi}} - \bar{u}_i \bar{S}_{upi} \tag{9}$$

If the drag force is again used as the momentum source term, as in Eq. (5), the following expression for the kinetic energy source term due to the presence of particles can be obtained after Reynolds averaging:

$$S_{kp} = \frac{\alpha \rho_p}{\tau_p}\left(|\bar{u}_i - \bar{u}_{pi}|^2 + \left(\overline{u'_{pi} u'_{pi}} - \overline{u'_i u'_{pi}}\right)\right) \tag{10}$$

The first term can be identified as the transfer of energy by the drag force while the last two terms is seen to represent the transfer of kinetic energy of the particle motion to the kinetic energy of the of the fluid. The first term is always positive and increases in magnitude with particle size for particles traveling at terminal velocity. According to Crowe (2000) the last two terms can be neglected for dilute flow but become important for dense flow where particle collisions tend to increase the particle kinetic energy. Thus models based on this consistent approach is only able to predict an augmentation of the carrier phase turbulence; the opposite of the standard approach. The source term to the dissipation rate is found similarly as for the standard approach, however, the value of the constant $C_{\varepsilon 3}$ should be changed to 1.8 (Lain and Sommerfeld, 2003). This value is often discussed and several observations suggest that this value is not universal (Squires and Eaton, 1992; Boulet and Moissette, 2002).

3.3. Semi-empirical or semi-heuristic approach

The third approach to formulate appropriate source terms to the turbulence equations deals with additional semi-empirical production and dissipation terms based on energy transfer mechanisms associated with the particles. The production of turbulence is most often attributed to the wake of the particle where the velocity defect and vortex shedding are well known phenomena which influence the carrier phase. Yuan and Michaelides (1992) and Yarin and Hetsroni (1994) have both presented models in which production terms rely on descriptions of the wake, while Kenning and Crowe (1997) introduces a hybrid length scale, in replacement of the traditional dissipation length scale to account for the additional dissipation. These models have succeeded in predicting some changes in the turbulence intensity but have been criticized for not providing a theoretical base consistent with the closures presented above (Boulet and Moissette, 2002).

3.4. New source term

The standard and the consistent approach are theoretical "correct" in that they both are derived considering the conservation of energy, but neither is fully capable of predicting both attenuation and augmentation of the fluid phase. Semi-empirical models use a mechanistic approach to formulate terms which with some success can account for both attenuation and augmentation, but these models are criticized for not being based on a solid theoretical basis. What is desired is thus a model which is derived on a theoretical basis but which contains both production and dissipation terms which can be related to fundamental mechanisms.

Referring to the comprehensive DNS study by Vreman (2007) for inspiration, two basic mechanisms can be identified as causes for turbulence modulation in pipe flows: One is due to the particles mean velocity profile generally being more uniform than the carrier phase mean velocity profile, and the other resulting from the particle-eddy interaction which leads to additional dissipation. The momentum source term can thus be extended to yield two simple forcing terms reflecting the basic mechanisms:

$$S_{upi} = S_{upi,1} + S_{upi,2} = \frac{\alpha\rho_p}{\tau_p}(u_{pi} - \bar{u}_i) + \frac{\alpha\rho_p}{\tau_p}(\bar{u}_i - u_i) \tag{11}$$

If the consistent approach is applied on the first term and the standard approach on the second, the source term due to particles can be expressed as:

$$S_{kp} = \frac{\alpha\rho_p}{\tau_p}(u_{pi} - \bar{u}_i)(u_{pi} - \bar{u}_i) + \frac{\alpha\rho_p}{\tau_p}u'_i(\bar{u}_i - u_i) \tag{12}$$

Performing Reynolds decomposition along with Reynolds averaging the final expression emerges as:

$$S_{kp} = \frac{\alpha\rho_p}{\tau_p}\left(|\bar{u}_i - \bar{u}_{pi}|^2 + \overline{u'_{pi} u'_{pi}} - 2k\right) \tag{13}$$

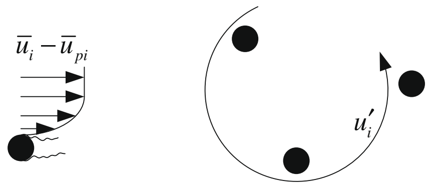

Fig. 1. Mechanisms for turbulence modulation.

This term can also be achieved by adding the source terms of the standard and consistent method and thus represent a combination of both approaches. The terms in Eq. (13) can be related to the two mechanisms for the transfer of mechanical energy of the particle phase to the turbulent kinetic energy of the fluid phase. These mechanisms are illustrated in Fig. 1.

Particles represent surfaces which are capable of supporting stresses and thus generate additional turbulence due to the flow gradient. This additional turbulence manifests itself in the wake of the particles and is often referred to as "wake induced" turbulence. This is also addressed by the consistent approach and is dependent on concentration, the relative velocity between the particle and the fluid phase which for particles traveling at terminal velocity is highly dependent on the particle size. This term reflects the conversion of mechanical work by the drag force and is thus related to the first mechanism.

The correlated motion between particles and turbulent eddies tend to attenuate the turbulence as the particles are accelerated by the fluid motion. This mechanism should be dependent on concentration, relevant turbulence quantities and the particle response time. This mechanism is also addressed by the standard approach but not the consistent approach and reflects the second mechanism.

The source term for the dissipation equation can again be found using Eq. (8) where the constant $C_{\varepsilon 3}$ is set initially to 1.0. Several different values of the proportionality constant between values of 1 and 2 have been tried, however, the effect on the final outcome is very limited and the initial value of 1.0 has been maintained. It can be realized that this derivation yields the desired effects relating to experimental observations. For small particles the first term will be small compared the third term and thus the overall effect of the source term is to attenuate turbulence. For large particles falling at terminal velocity the first term will be dominant and source term will thus be able to reproduce the large augmentation which has been observed.

3.5. Simple closures to test the new source term

The suggested equation for the turbulence kinetic energy budget for particle-laden flows now appear:

$$\rho\frac{\partial k}{\partial t}+\rho\bar{u}_j\frac{\partial k}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\mu+\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right)-\rho\overline{u_i'u_j'}\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\alpha\rho_p}{\tau_p}\left(|\bar{u}_i-\bar{u}_{pi}|^2+\overline{u_{pi}'u_{pi}'-2k}\right)-\rho\varepsilon \quad (14)$$

When this equation is applied to the thought experiment by Crowe (2000), where particles are artificially fixed in position in an otherwise steady and uniform flow,[2] Eq. (14) is reduced to the following: (See Fig. 2)

$$\frac{\alpha\rho_p}{\tau_p}\left(|\bar{u}_i|^2-2k\right)-\rho\varepsilon=0 \quad (15)$$

[2] A flow with no spatial or temporal gradients in the averaged properties. This represents an ideal case which can be used to compare models.

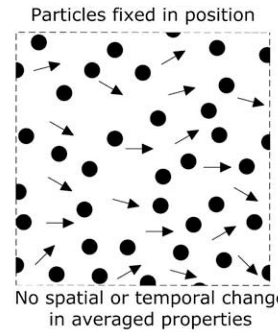

Fig. 2. Schematic of the artificial flow where particles are fixed in position developed by Crowe (2000) for test of turbulence modulation models.

which states that the turbulence produced by the particles is dissipated by the combined dissipative effect of the particles and viscosity. Thus the modeled source term is consistent in the way that it provides a plausible closure for this idealized flow.

Another simple closure for the turbulence modulation at the centerline of a pipe also presented by Crowe (2000) may also serve to evaluate this new term. When applied to the case of a fully developed dilute particle-laden flow in a vertical pipe, for which experimental data is available, Eq. (14) for the flow near the pipe centerline is reduced to:

$$-\rho\overline{u_i'u_j'}\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\alpha\rho_p}{\tau_p}\left(|\bar{u}_i-\bar{u}_{pi}|^2-2k\right)-\rho\varepsilon=0 \quad (16)$$

Here it has been assumed that the particle kinetic energy is negligible at the centerline of the pipe. This assumption is addressed later in the paper. Using the closure scheme provided by Crowe (2000) where the terminal velocity $g\tau_V/f$ is used for the velocity difference, Eq. (16) reduces to:

$$\rho\frac{k_0^{3/2}}{l_e}+\frac{\alpha\rho_p f}{\tau_V}\left(\left(\frac{g\tau_V}{f}\right)^2-2k\right)-\rho\frac{k^{3/2}}{l_e}=0 \quad (17)$$

where respectively, k and k_0 are the turbulent kinetic energy of the clear flow and particle-laden flow and l_e is the integral length scale. Notice that unlike the work by Crowe (2000) it is not necessary to resort to a "hybrid" length scale. Without using the hybrid length scale in the closure scheme the model suggested by Crowe (2000) is only able to predict augmentation. However, the use of the hybrid length scale produces obviously erroneous results for low particle volume fractions. Here, the following supporting equations are used:

$$f^{5/2}=0.058\frac{g\tau_V d_p\rho}{\mu},\qquad \tau_V=\frac{\rho_p d_p^2}{18\mu}$$
$$\alpha=\left(\frac{\rho}{\rho_p}\right)C,\quad k_0=\tfrac{3}{2}(\bar{u}\sigma_0)^2,\quad k=\tfrac{3}{2}(\bar{u}\sigma)^2 \quad (18)$$

where respectively, σ and σ_0 is the turbulence intensity of the clear and particle-laden flow. The correlation for f is an approximation which is valid for particles traveling at terminal velocity (Crowe, 2000). The fractional change of the turbulence intensity for a pipe flow with mean velocity of 10 m/s laden with glass particles in a 40 mm pipe and a particle free turbulence intensity of 0.06, have been solved using an iterative procedure. The turbulence length scale at the center of the pipe is set at l_e = 4 mm (Hutchinson et al., 1971). The particle size has been varied for particle mass concentrations of 0.1, 1 and 5 producing the curves seen in Fig. 3.

Similar to the model by Crowe (2000) the curves show the same trends as the experimental measurements. Furthermore, this model also predicts the correct behavior when approaching the one-way coupling regime.

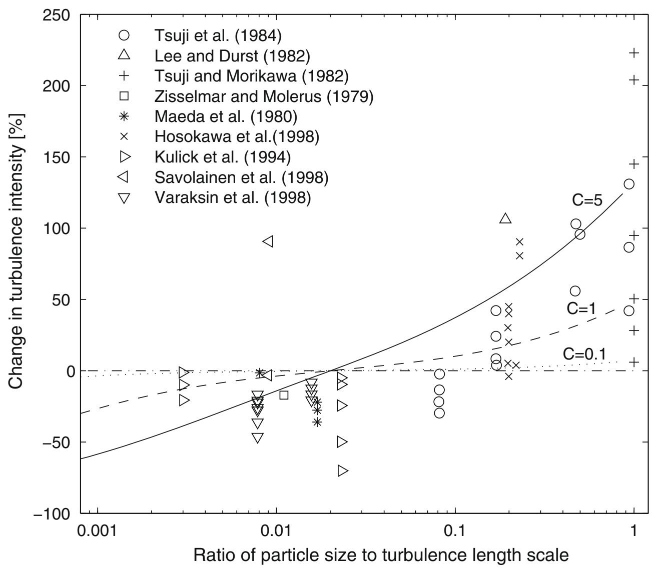

Fig. 3. Comparison of model prediction and data for the turbulence modulation at the centerline of a vertical pipe.

Making the pipe in the model smaller will generally move the curves towards the right on the *x*-axis while using a larger pipe will move them to the left. Using relative velocities other than the terminal velocity can similar dramatically change the prediction due to the sensitivity of the source term. In this closure scheme only the effect on the k-equation is considered whereas it is known that the effect of the momentum coupling tend to decrease the turbulence intensity further. Finally, the double correlation of the particles fluctuating velocity, which is neglected here, becomes important for dense flows in particular but also for wall bounded flows (Vreman, 2007).

The three models evaluated in this paper have been applied on the same simple closure scheme as presented above. Table 2 summarizes the source terms used in the evaluation of the different approaches. Note that some terms have been neglected for simplicity.

Table 2
Source terms evaluated in present investigation.

Standard:	$S_{kp} = \frac{\alpha\rho_p}{\tau_p}\left(2k\frac{\tau^*}{\tau^*+\tau_p} - 2k\right)$
Consistent:	$S_{kp} = \frac{\alpha\rho_p}{\tau_p}\lvert\bar{u}_i - \bar{u}_{pi}\rvert^2$
New:	$S_{kp} = \frac{\alpha\rho_p}{\tau_p}\left(\lvert\bar{u}_i - \bar{u}_{pi}\rvert^2 - 2k\right)$

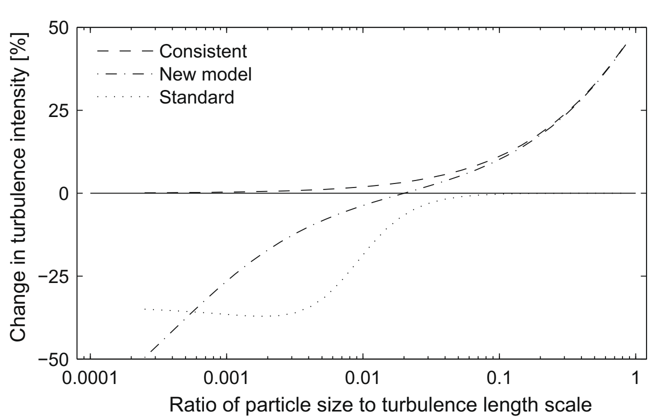

Fig. 4. Comparison of the consistent, the new model and the standard model evaluated at $C = 1$.

Fig. 4 shows the result for the same 40 mm pipe where the particle size is varied between 10 and 4000 μm with unity loading. It can be seen that the consistent model is only able to predict augmentation whereas the standard model, here represented by the model by Lightstone and Hodgson (2004) is only able to predict attenuation.

The new model, which essentially is a hybrid between the standard and consistent approach, is able to predict both augmentation and attenuation. For very large particle sizes the particle-eddy interaction mechanism diminishes and the evaluation of the consistent and the new model becomes the same while the standard model predicts zero turbulence modulation. For small particles the consistent approach predicts zero modulation whereas both the standard and new model predicts significant attenuation.

4. Results

On the basis of the preliminary evaluation of the new source term based on simple closure schemes, three cases have been chosen to evaluate the performance of the new source term in the k–ε framework. Case 1 consists of the largest particles where significant augmentation is expected while Case 3 comprises small particles where attenuation is expected. Case 2 encompass "medium" sized particles which have a d_p/l_e ratio close to the criterion defined by Gore and Crowe (1989) which marks the boundary between attenuation and augmentation and thus very little modification of the carrier phase is to be expected. Cases 1 and 2 is taken from the experimental study by Tsuji et al. (1984) while case 3 is taken from Kulick et al. (1994). Both studies deals with the air-particle flow in a vertical pipe, where Laser Doppler Velocimetry has been used to measure the carrier phase velocity in the axial direction. Experimental results are available for a range of different pipe Reynolds numbers, particle mass loadings and particle diameters. The details of the experimental settings are shown in Table 3.

Besides the differences in flow rate, pipe diameter and particle materials it should be noted that the study Tsuji et al. is an upward flow whereas the study by Kulick et al. is a downward flow. Results for all cases are given at 5 m from the inlet where the flow can be considered to be fully developed. Polystyrene (cases 1&2) and copper (case 3) particles are used which yields a density ratio of around or above 1/1000.

According to the guidelines provided in (Sommerfeld et al., 2007) the influence of added mass, Basset history force and pressure gradient is negligible for the motion of the particles. Only loadings for which the flow can be considered as dilute (Elghobashi, 1994) are used and particle collisions can thus be neglected. The pipe used in the experiments were made of glass; thus the pipe wall can be considered as being smooth and particle-wall collisions are assumed to be perfectly elastic for the no slip wall boundary. The calculations have been performed on a two dimensional axisymmetric mesh discretised with 20×800 (case 1 and 2) and 30×800 (case 3) control volumes in the radial and axial directions

Table 3
Test cases.

	Case 1	Case 2	Case 3
d_p (μm)	1420	243	70
ρ (kg/m^3)	1030	1020	8800
Loading	0.6	0.5	0.4
D_{pipe} (mm)	30.5	30.5	40.0
$u_{c,centerline}$ (m/s)	13.4	13.4	10.5
u_{mean} (m/s)	11.26	11.26	8.85
$\dot{m}_p$ (kg/s)	0.00605	0.00504	0.00545
d_p/l_e *	0.47	0.08	0.02

* Evaluated at centerline: $l_e = 0.1D$.

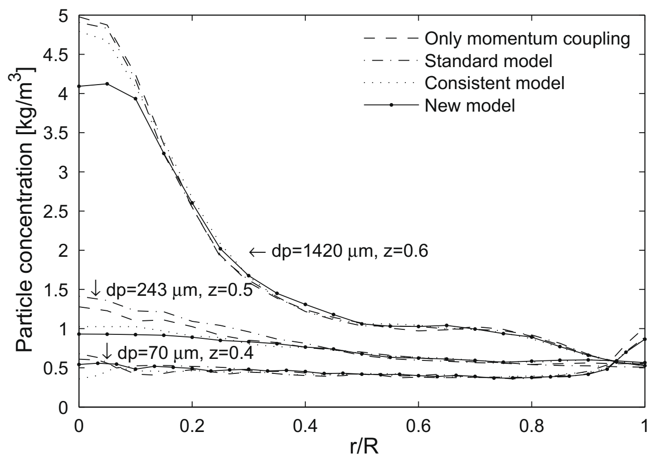

Fig. 5. Concentration profiles of the three different cases for the three models and simulations with only momentum coupling.

respectively. This mesh has been proven to produce grid independent results. At the inlet a top hat velocity profile has been specified and the initial velocities for the particles are set equal to the gas phase. A total of 25,000 particle trajectories have been simulated to provide statistically independent results. At the pipe exit zero gauge pressure has been specified across the entire boundary and the particles are allowed to escape.

Fig. 5 shows the concentration profiles for the three cases and the different models considered.

It can be seen that the largest particles are concentrated towards the center of the pipe whereas the concentration profile for the smallest particles is mostly evenly distributed except close to the wall where particles have accumulated. Since the source terms for all models considered is proportional to the particle concentration it can be concluded that for large particles the numerical values of the source terms are several magnitudes larger at the pipe center than in the near wall region. For cases 1 and 2 there is a clear coupling between the source term formulation and the particle concentration whereas for the smallest particles a flat particle concentration profile can be assumed for all source terms.

Fig. 6 comprises the results from the different models for case 1 and also shows the measurements by Tsuji et al. (1984) for this case. For the mean velocity all models tend to flatten the velocity profile slightly. This effect is most pronounced for the new model and for the model by Lightstone and Hodgson (2004). This effect is however difficult to perceive in the experimental data where the difference between the clear flow and particle-laden profiles are minimal. Furthermore it can be noticed that it is not possible to reproduce the clear flow velocity profile exactly, a problem which also can be found in other investigations dealing with the numerical simulation of this case (Lain and Sommerfeld, 2003; Yan et al., 2007). For the fluctuating velocity component the model by Lightstone and Hodgson and the simulation using momentum coupling only predict an attenuation of the flow while both the new model and the model by Lain and Sommerfeld predicts augmentation of the turbulence. It can be noticed at the new model performs slightly better than the model by Lain and Sommerfeld. Again it should be noted that it is not possible to predict the exact same clear flow profile as measured by Tsuji et al. This is to a part due to the Boussinesq approximation, fundamental to the k–ε model, which treats the turbulence as being isotropic. Thus the fluctuating velocity u' is calculated as $\sqrt{\frac{2}{3}k}$.

Fig. 7 comprises the results of the numerical simulations and the measurements by Tsuji et al. (1984) for case 2. For all the tested models the mean velocity profiles for this case is almost indistinguishable from the clear flow profile whereas for the measurements the particle-laden profile is somewhat flatter than the clear flow profile and similarly the measurements of the fluctuating velocity component is dampened compared to the clear flow profile. The prediction for the standard approach as well as the prediction with the momentum source term only display the same

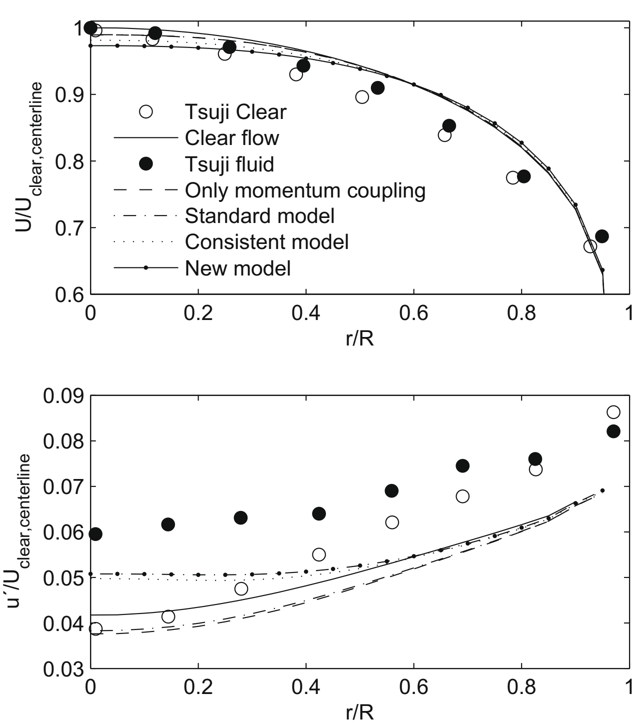

Fig. 6. Non-dimensional radial profiles of the axial mean and fluctuating velocity components for case 1: $dp = 1420$ μm, $z = 0.6$, $U_{clear,centerline} = 13.4$ m/s.

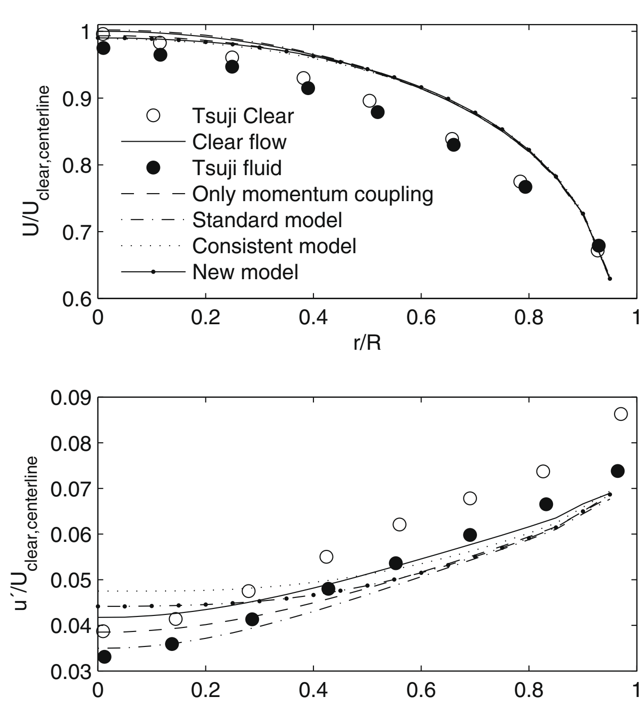

Fig. 7. Non-dimensional radial profiles of the axial mean and fluctuating velocity components for case 2: $dp = 243$ μm, $z = 0.5$, $U_{clear,centerline} = 13.4$ m/s.

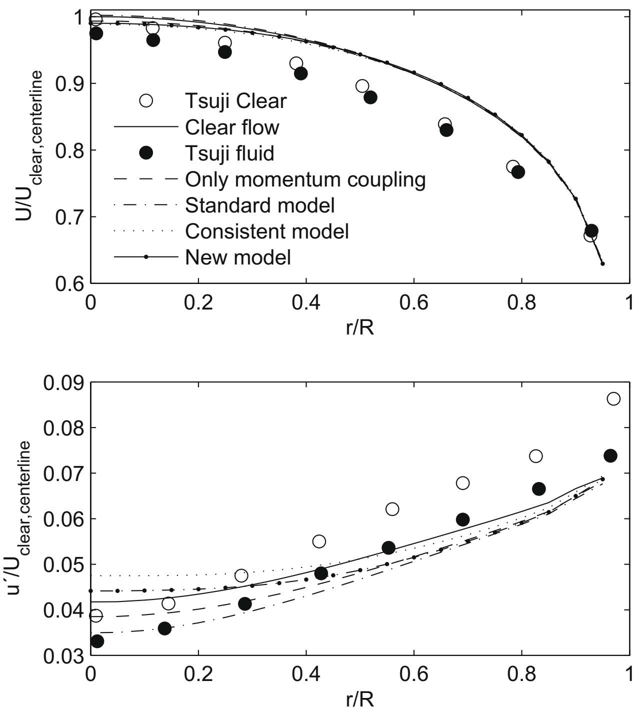

Fig. 8. Non-dimensional radial profiles of the axial mean and fluctuating velocity components for case 3: $dp = 70\ \mu m$, $z = 0.4$, $U_{clear,centerline} = 10.5$ m/s. For the mean velocity profile the experimental results by Kulick et al. (1994) for the clear flow is indistinguishable from the particle-laden flow.

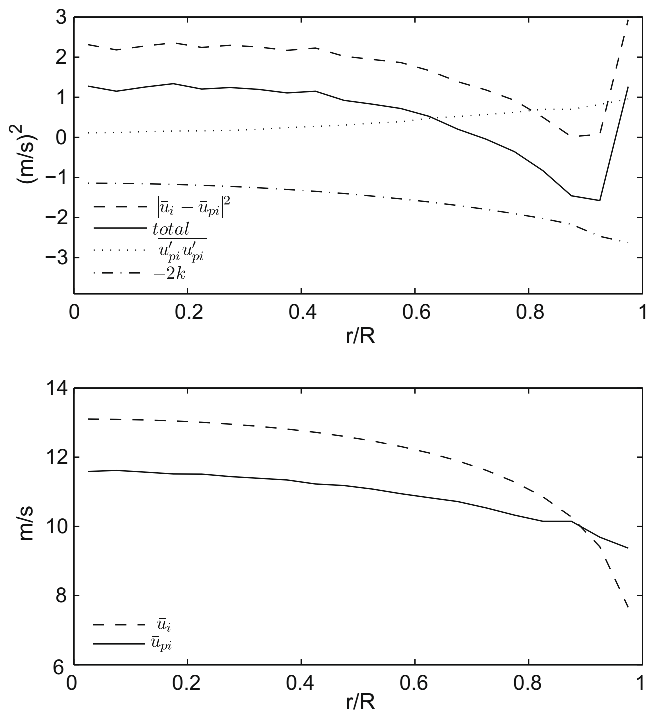

Fig. 9. Evaluation of the different terms in the new model and mean flow velocity profiles for case 2.

trend as the measurements whereas the new model and the consistent approach predicts an augmentation of the carrier phase. For this case the standard approach provides the best approximation to the experimental data at the centerline. It can be noticed that the new model performs better then the consistent approach and the model predicts only a relative small change at the centerline which can be expected on basis of the d_p/l_e ratio.

Fig. 8 comprises the results of the numerical simulation for case 3 which is compared to the experimental results by Kulick et al. (1994). The measurement of the mean velocity profile for the clear flow is indistinguishable from that of the particle-laden flow. For the prediction of the mean velocity profile there is similarly hardly any difference between the clear flow and the prediction by the different models. For the fluctuating flow all models now predicts attenuation of the carrier phase at the centerline. For this case the relative velocity which play an essential role in predicting the augmentation caused by larger particles is relatively small and thus for the consistent model only the effect by the momentum coupling is causing the attenuation. For the new model and the standard model additional terms exists which caused the prediction to become less than that caused by the momentum coupling alone.

Fig. 9 shows an evaluation of the different terms in the new model for case 2. It can be seen that both attenuation and augmentation present at different regions of the flow. The relative velocity approaches zero in a region of the flow since the mean particle velocity is more uniform than the mean fluid velocity. In the region where the relative velocity is small there is significant attenuation of the fluid turbulence. Close to the wall the relative velocity increases rapidly and thus there is significant augmentation in the near wall also for small and heavy particles. At the center of the pipe the square of the relative velocity is larger then the twice the turbulent kinetic energy and the resultant evaluation of the entire source term is thus positive. At the centerline $\overline{u'_{pi}u'_{pi}}$ is an order of magnitude smaller than the other terms and can be neglected. However, this term increases in magnitude closer to the wall due to the particle-wall collisions and always acts as a source. Furthermore, it can be realized the often used assumption where the relative velocity is approximated by the terminal velocity is only valid in the center of the pipe.

5. Discussion and conclusion

In this paper, the performance of a new Lagrangian formulation for the source terms to the turbulence equations has been evaluated and compared with results obtained with the so-called standard and consistent approach as well as with experimental measurements. Previous attempts to model the entire range of turbulence modulation experiments have been only partially successful and suffered from relying on semi-heuristic/empirical formulation which is not consistent with the governing equations of fluid flow. Up to now only the standard and the consistent approach is derived directly from the governing equations for particle-laden flow. The standard approach is only able to predict attenuation whereas the consistent approach only contains mechanisms which enhance the turbulence. The new model relies on a new derivation, consistent with the balance equations, to formulate terms which contains mechanisms for both the suppression and enhancement of turbulence and the new model can be seen as a combination of the standard and the consistent terms. Furthermore, no additional modeling is necessary for the new model since particle and fluid kinetic energy is given explicitly. The performance of the new model surpass that of both the standard and the consistent model for the present range of investigations, however before a stronger conclusion may be stated it is necessary to test this model on other flow situations and for other closure schemes such as the Reynolds Stress Model.

Acknowledgement

This work was supported by Energinet.dk, under Grant PSO 2006 1 6364.

References

Boulet, P., Moissette, S., 2002. Influence of the particle-turbulence modulation modelling in the simulation of a non-isothermal gas–solid flow. International Journal of Heat and Mass Transfer 45 (20), 4201–4216.

Chen, P.E., Wood, C.E., 1985. A turbulence closure model for dilute gas-particle flows. Canadian Journal of Chemical Engineering 63, 349–360.

Crowe, C.T., 2000. On models for turbulence modulation in fluid-particle flows. International Journal of Multiphase Flow 26 (5), 719–727.

Crowe, C.T., Sharma, M.P., Stock, D.E., 1977. Particle-source in cell (Psi-Cell) model for gas-droplet flows. Journal of Fluids Engineering – Transactions of the ASME 99 (2), 325–332.

Elghobashi, S., 1994. On predicting particle-laden turbulent flows. Applied Scientific Research 52 (4), 309–329.

Fluent Inc. 2006, Fluent 6.3 User's Guide.

Gore, R.A., Crowe, C.T., 1989. Effect of particle-size on modulating turbulent intensity. International Journal of Multiphase Flow 15 (2), 279–285.

Gouesbet, G., Berlemont, A., 1999. Eulerian and Lagrangian approaches for predicting the behaviour of discrete particles in turbulent flows. Progress in Energy and Combustion Science 25 (2), 133–159.

Hutchinson, P., Hewitt, G.F., Dukler, A.E., 1971. Deposition of liquid or solid dispersions from turbulent gas stream – stochastic model. Chemical Engineering Science 26 (3), 419.

Kenning, V.M., 1996. Self-induced turbulence in solid-liquid flow, Ph.D. Thesis, Washington State University.

Kenning, V.M., Crowe, C.T., 1997. On the effect of particles on carrier phase turbulence in gas-particle flows. International Journal of Multiphase Flow 23 (2), 403–408.

Kulick, J.D., Fessler, J.R., Eaton, J.K., 1994. Particle response and turbulence modification in fully-developed channel flow. Journal of Fluid Mechanics 277, 109–134.

Lain, S., Sommerfeld, M., 2003. Turbulence modulation in dispersed two-phase flow laden with solids from a Lagrangian perspective. International Journal of Heat and Fluid Flow 24 (4), 616–625.

Lightstone, M.F., Hodgson, S.M., 2004. Turbulence modulation in gas-particle flows: a comparison of selected models. Canadian Journal of Chemical Engineering 82 (2), 209–219.

Sommerfeld, M., van Wachem, B., Oliemans, R. (Eds.), 2007, Best Practice Guidelines – on Computational Multiphase Dynamics for Turbulent Dispersed Multiphase Flows, Version 16-oct-07 edn, ERCOFTAC Special Interest Group on Dispersed Turbulent Multi-Phase Flow.

Squires, K.D., Eaton, J.K. 1992. On the modelling of particle-laden turbulent flows, In: Proceedings of the 6th Workshop on Two-phase Flow Predictions.

Tsuji, Y., Morikawa, Y., Shiomi, H., 1984. LDV measurements of an air solid 2-phase flow in a vertical pipe. Journal of Fluid Mechanics 139 (FEB), 417–434.

Vreman, A.W., 2007. Turbulence characteristics of particle-laden pipe flow. Journal of Fluid Mechanics 584, 235–279.

Yan, F., Lightstone, M.F., Wood, P.E., 2007. Numerical study on turbulence modulation in gas-particle flows. Heat and Mass Transfer 43, 243–253.

Yarin, L.P., Hetsroni, G., 1994. Turbulence intensity in dilute 2-phase flows 3. The particles-turbulence interaction in dilute 2-phase flow. International Journal of Multiphase Flow 20 (1), 27–44.

Yuan, Z., Michaelides, E.E., 1992. Turbulence modulation in particulate flows – a theoretical approach. International Journal of Multiphase Flow 18 (5), 779–785.